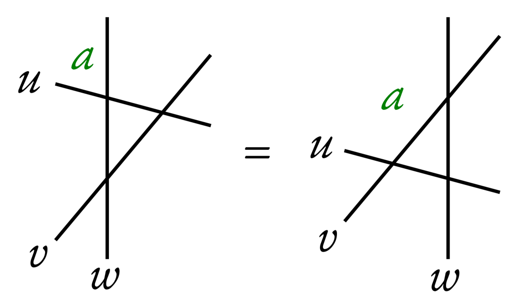
The constructive method for the elliptic solid-on-solid model with domain walls and a reflecting end

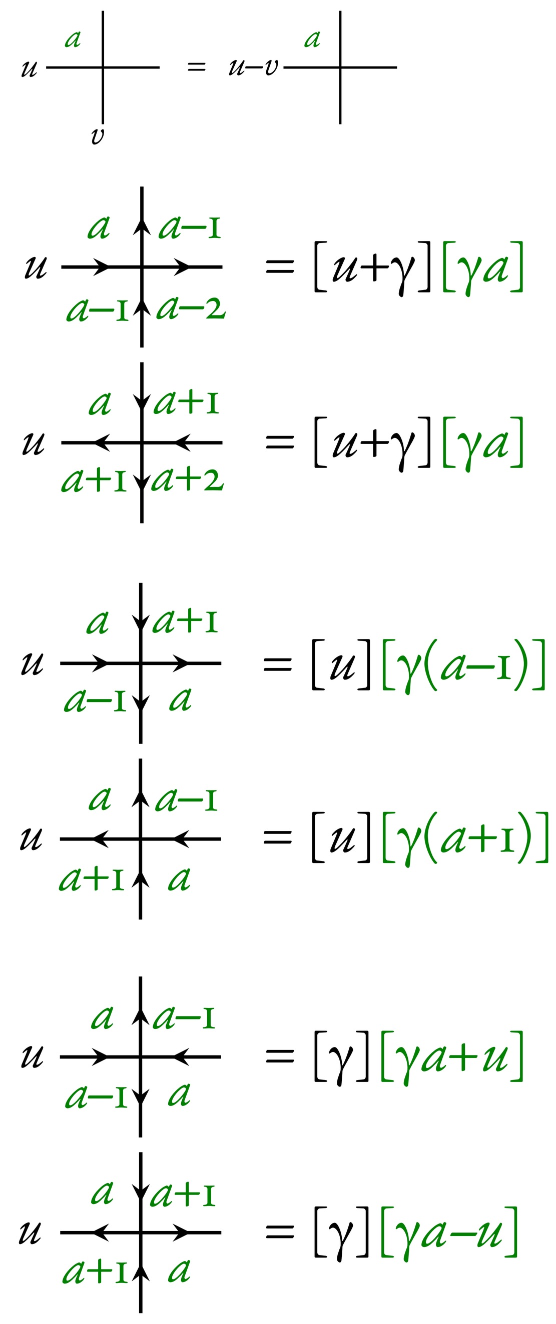
Mix and match: choose your favourite example

Six-vertex model, $[u] = \sin u$

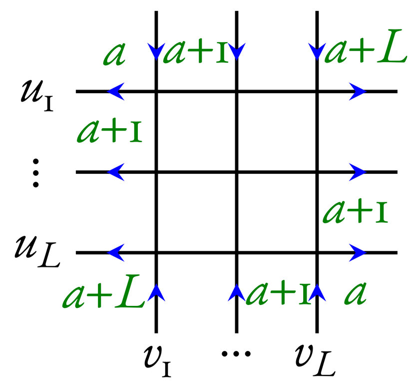
option: upgrade to solid-on-solid (height) model — Baxter '72

option: upgrade to elliptic case, $[u] = \theta(u\,;p)$



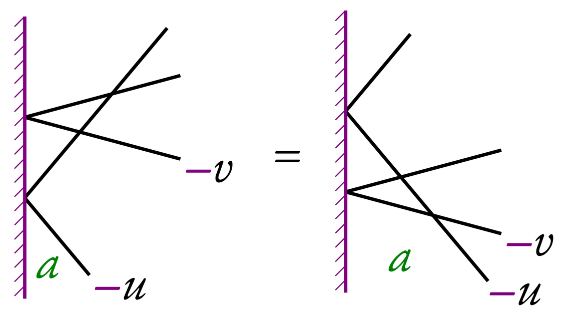


with domain-wall boundary conditions

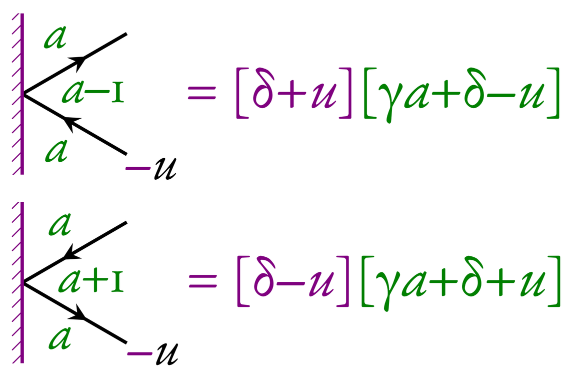


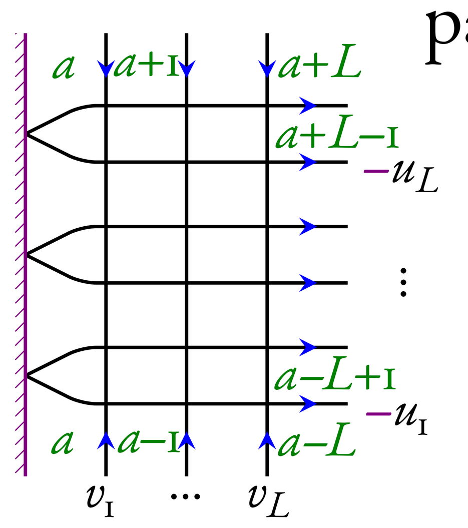
Korepin '82
Izergin '87
Rosengren '09

option: include one reflecting end



Cherednik '84, cf. Sklyanin '88
Behrend–Pearce–O'Brien '96





Goal: compute the partition function Z_L

Tsuchiya '98
Filali–Kitanine '10
Filali '11

The constructive method

- contains the Korepin–Izergin approach
- provides a *recipe* for constructing the solution
- can be made rigorous, cf. the six-vertex model with DWBCs
- extends to the elliptic SOS model with DWBCs and a reflecting end

Korepin's characterization

There exists a unique $\{Z_L\}_{L\geq 1}$ such that

- *recursion* in L: $Z_L|_{u_L=v_L-\gamma} = \text{factor} \times Z_{L-1}$ *
- initial condition: $Z_{L=1} = [\gamma][\gamma a - u]$
- symmetric function in u_i *
- symmetric function in v_j
- crossing symmetry *
- in certain function space
- certain degree in variables *

Korepin '82
Tsuchiya '98
Rosengren '09
Filali–Kitanine '10
Filali '11

* JL '16
* Galleas '12
* JL '15
* JL '15 '16

Constructive method

Galleas '10 '11 '12 '13
Galleas–JL '14
JL '15 '16

There exists a unique $\{Z_L\}_{L\geq 1}$ such that

- linear *functional equation* at fixed L:

  $Z_L = \Sigma_i \,(\text{explicit coeff})_i \times Z_L|_{\text{omit } u_i,\text{ include } u_0}$
- fixed value at some point for each Z_L
- in certain function space

e.g. trigonometric polynomials
higher-order theta functions

Galleas '11

Izergin's solution

$$Z_L \sim \det_{i,j}\left([\gamma]/[u_i-v_j][u_i-v_j+\gamma]\right) \,/\, \det_{i,j}\left(1/[u_i-v_j]\right)$$

Izergin '87
Tsuchiya '98
Rosengren '09
Filali–Kitanine '10
Filali '11

cf. JL '16

Get solution in form of sum over permutations

cf. Baxter '87

more info?
- arXiv:1510.00342
- PhD thesis '16
- upcoming review
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